RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

FIRST YEAR [2017-20]

B.A./B.Sc. FIRST SEMESTER (July – December) 2017

Mid-Semester Examination, September 2017

Date : 15/09/2017 **MATHEMATICS (General)**

Time : 12 noon – 1 pm Paper : I Full Marks : 25

[Use a separate Answer Book for each group]

Group – A

Answer **any one** of **Question No. 1 & 2** : [1×4]

1. a) If z_1, z_2 are complex numbers such that $|z_1 - 5z_2| = |5 - z_1\bar{z}_2|$ and $|z_2| \neq 1$, prove that $|z_1| = 5$. [2]

   b) Find the principal value of $(1+i)^i$. [2]

2. a) Using De Moivre's theorem prove that $\tan 4\theta = \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$. [2]

   b) Find all the values of z such that $\cos(z-1) = 0$. [2]

3. Answer **any one** of the following : [1×4]

   a) Prove that $\begin{vmatrix} x+1 & 2 & 2 \\ 2 & x+1 & 2 \\ 2 & 2 & x+1 \end{vmatrix} = (x-1)^2(x+5)$.

   b) Prove that $\begin{vmatrix} a^3+1 & a^2+2 & 1 \\ b^3+1 & b^2+2 & 1 \\ c^3+1 & c^2+2 & 1 \end{vmatrix} = \begin{vmatrix} a^2 & a & bc \\ b^2 & b & ca \\ c^2 & c & ab \end{vmatrix}$.

Group – B

4. Answer **any two** of the following : [2×4]

   a) Let A, B, C be subsets of a universal set S. Prove that $A \times (B \setminus C) = (A \times B) \setminus (A \times C)$.

   b) Define a relation ρ on $\mathbb{Z}$ by setting "$a \rho b$ iff (a – b) is divisible by 3". Check if ρ is an equivalence relation.

   c) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 3x + 2$. Show that f is invertible. Find f^{-1}.

   d) Let $f : A \to B$, $g : B \to C$ be surjective maps. Show that $gof : A \to C$ is surjective.

Group – C

4. Check whether $\sqrt{5}$ is a rational or not. [3]

5. Answer **any two** of the following : [2×3]

   a) Using $\in -\delta$ definition find $\lim\limits_{x\to\infty} \dfrac{\sin x}{x}$.

b) A function f is defined by

$f(x) = -x$ when $x \leq 0$

$= x$ when $0 < x < 1$

$= 2 - x$ when $x \geq 1$

Show that f is continuous at x = 0 and x = 1.

c) $f(x) = \frac{x^2 - 16}{x - 4}$. What value must be assigned to f(4), if f(x) is to be continuous at x = 4.

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